

ORIGINAL RESEARCH ARTICLE

Mathematic modeling of the dose in water for calculating collimator scatter factor of flattening filter and flattening filter free megavoltage photon beams

Supplementary File

S1. The derivation of percent depth dose (PDD) using the buildup-tail model in regular and irregular field size of high-energy photon beam.

The PDD for four high-energy photon energies of 4, 6, 10, and 18 MV was modeled in this study. The measurements data were for 6 and 10 MV while 4 and 18 MV were the published data from previous study^[1]. The data were used for testing the power of the mathematic modeling proposed in this study. The results of modeled PDD of 4 and 18 MV are demonstrated in the [Figure S1](#) and no flattening filter (FF)/FF free (FFF) analysis in the contains.

The converted curves of PDD of high-energy photon beams measured by the ion chamber were coincident with film measurements. The PDD with different energies adopted in this study was already measured by the water phantom at the commission and was compared by the experimental measurements in this study. By adjusting the main parameters of n and μ , the best fitting for four photon PDD curves in every energy at the field sizes of 10 cm × 10 cm. [Figure S1](#) shows the fitting results of PDD curves by the buildup-tail function of published data of photon energies of 4 and 18 MV in regular field size, while [Figure S2](#) shows the results of measured photon energies for 6 and 10 MV in regular and irregular field sizes.

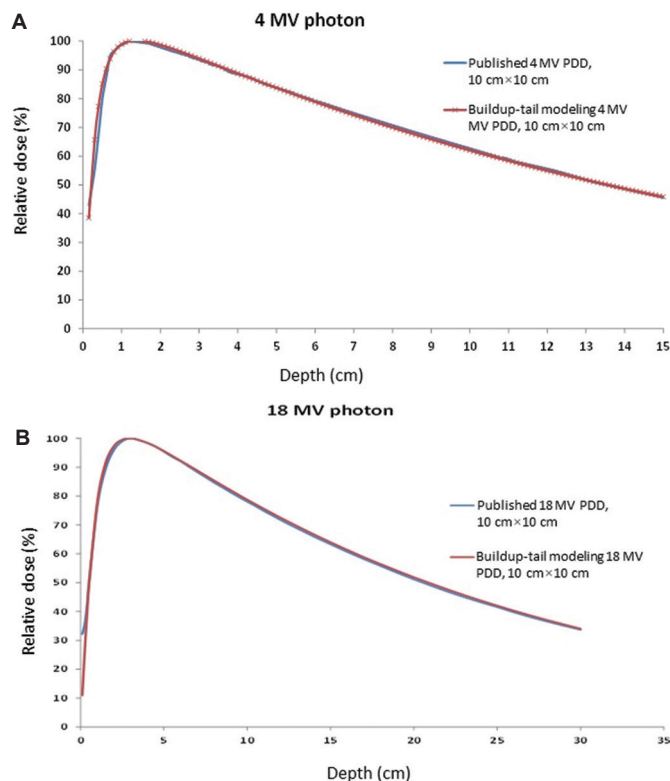


Figure S1. The best fitting of photon percent depth dose curves at energies (A) 4 and (B) 18 MV modeled from the published data.

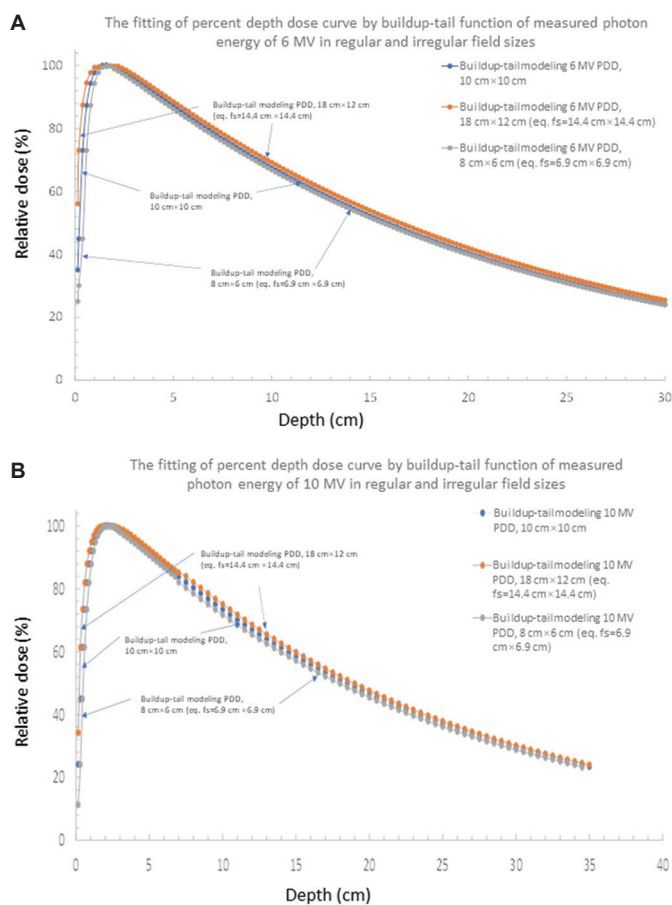


Figure S2. The best fitting of photon percent depth dose curves at energies (A) 6 and (B) 10 MV modeled from the experimental measured data.

S2. The derivation of buildup-tail function in determining the Sc of FF and FFF in high-energy photon beam.

To enable a better understanding of the origin of the home-generated buildup-tail function, the author describes the derivation of this function in the following.

The buildup-tail model originated from the proportion function $y(x) = \frac{1}{x}$. When x increases from $-\infty$ to 0, the curve of y is located in the region of $(-, -)$ quadrant. When x goes from 0 to $+\infty$, the curve of y is located in the region of $(+, +)$ quadrant. Let $\frac{1}{x}$ be $\frac{1}{|x|}$, then the curve of $y(x)$ falls in the $(-, +)$ and the $(+, +)$ quadrants. The curve of $y(x) = \frac{1}{x}$ has a left and right tail of dose-profile-shape pattern. Let $y(x) = \frac{1}{|x|} = \frac{1}{\sqrt{(x^2)}}$. When $x=0$, $y(x)$ becomes infinite, which does not happen in real dose-profiles. Therefore, we insert n into $y(x)$ to be: $f(x) = \left(\frac{1}{\sqrt{n+(x^2)}} \right)$, where $n > 0$, let $tail(x) = \frac{x}{\sqrt{n+x^2}}$, where x is the depth in water in unit cm, n is a scalar of the spread factor in real numbers.

On the other hand, the function $tail(x) = \frac{x}{\sqrt{n+x^2}}$ demonstrates an ascending value $tail(x)$ with an increasing depth of x in water. When introducing an exponential function $e^{-\mu x}$ to $tail(x)$, the combination becomes PDD_{b-t} function $\frac{x}{\sqrt{x^2+n}} \cdot e^{-\mu x}$, namely PDD_{b-t} = \frac{d}{\sqrt{d^2+n}} \cdot e^{-\mu x}, where d is the depth in water in unit cm, $n > 0$, and is the harden factor of real number scalar.}

When $x = 0$, n plays an important role in avoiding $primary(x)$ to become infinite, while μ is the linear attenuation factor to fine turn the growth of the $\frac{x}{\sqrt{x^2 + n}}$ value.

Finally, the buildup-tail model can be expressed as follows:

$$PDD_{b-t} = \frac{d}{\sqrt{d^2 + n}} \cdot e^{-\mu d}$$

Reference

1. Padilla-Cabal F, Pérez-Liva M, Lara E, *et al.*, 2015, Monte Carlo calculations of an Elekta Precise SL-25 photon beam model. *J Radiother Pract*, 14: 311–322.
<https://doi.org/10.1017/S146039691500014X>