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A simulation analysis of the longer-term effects of immigration on per capita income in an aging population

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Abstract: Immigration is a possible instrument for offsetting longer-run adverse effects of population aging on per capita income. Our "laboratory" is a fictitious country Alpha to which we assign demographic characteristics typical of a country experiencing population aging. Simulations indicate that a very high immigration rate with heavy concentration in younger working ages might be required to keep per capita income from declining. More rapid productivity growth would also offset population aging as would higher rates of labour participation of older people. Longer life expectancy, taken alone, would lower per capita real income, as would higher fertility rates.

Keywords: immigration, per capita income, population aging, age structure, simulation

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1. Introduction

The dominant inducement for immigration policy today in many industrialized countries is population aging — a shift in age structure toward older ages brought about, during the second half of the 20th century, by a sequence of high fertility rates followed by declining and then persistently low rates (see Bongaarts, 1999, on the history of fertility rates), and continuing increases in life expectancy. This is the sequence that has resulted in the prospect of a large proportionate increase in the retired population, a concomitant decrease in the labour force proportion, and downward pressure on the level of income per capita. The prospect of population aging is widespread among industrialized countries (Anderson and Hussey, 2000). The effects will come sooner and be more pronounced for some countries, later and less pronounced for others, but the changes in age structure and demographic outlook are similar in the main, if not in the details and timing.

The phenomenon of population aging has been recognized for many years by demographers, economists, and others and there has been a variety of approaches used to assess the possible role of immigration as an instrument to offset its negative effects. Attention was given by various authors to population size and age distribution (Bijak, Kupiszewska, Kupiszewski *et al*., 2007, 2008; Loichinger, 2015; Mamolo and Scherbov, 2009; United Nations, 2013), the overall level of economic activity and standard of living (Barrell, Fitzgerald and Riley, 2010; Denton and Spencer, 2000; Kahanec and Zimmerman, 2008; Lee and Mason, 2011; Masson and Tryon, 1990), the fiscal positions of governments (Auerbach and Oreopoulos, 2000; Bonin, Raffelhuschen, Walliser *et al*., 2000; Dustmann, Frattini, Halls *et al*., 2010; Lee and Miller, 2000; Rowthorn, 2008; Storesletten, 2000), and more particularly to the sustainability of publicly-financed pension and health care programs (Alonso, 2009; Anderson and Hussey, 2000; Scherbov, Sanderson, Mamolo *et al*., 2014). Others have been concerned with possible undesirable effects of immigration on the incomes and employment of the domestic population (Borjas, 2003; Brücker and Jahn, 2009; Card, 2009, 2012; Dustmann, Fabbri and Preston, 2005, 2013; Friedberg and Hunt, 1995; Jean and Jiménez, 2010; Longhi *et al*., 2005; Okkerse, 2008; Ottaviano and Peri, 2007; Peri, 2012; Ruhs and Vargas-Silva, 2014) and on the distribution of government transfer payments as between immigrants and non-immigrants (Blanchflower and Shadforth, 2009; Kerr and Kerr, 2011).

Much of the economic literature on immigration and aging has been concerned with shorter or medium-term effects. Our paper on the other hand is concerned with the longer term, in particular the longer-term effects of immigration on a host country's national income per capita. Immigrants of working age increase the size of the labour force and add to the national product. But they and their dependents add also to the overall population, viewed as consumers, and thus affect both the numerator and denominator in the per capita calculation. Moreover, once in the country, immigrants have dependent children, age, and eventually themselves become elderly dependents. The effects on the host country's average income level, especially longer-term effects, may not be at all obvious without taking into account the demographic dynamics of immigration and its interaction with the host country's population.

We construct a theoretical model for a country with an aging population and assume an infinite supply of potential immigrants. The country, which we call Alpha, is fictitious and generic. We model its income generating process in as simple a fashion as possible for our purposes, calibrate the model, and use it in a series of simulation experiments in which we consider alternative immigration strategies and related issues of productivity, fertility rates, mortality reductions, and labour force participation rates of the older population. We use actual data as a basis for calibration but emphasize that the model is *theoretical*; it does not represent any actual country but in broad terms shares the demographic characteristics of many industrialized countries.

2. Methods

2.1 The Setting

The mythical country of Alpha is our "laboratory". For simplicity, the population of Alpha is divided into five broad age groups, corresponding to intervals of 20 years: Children (0–19), Young Adults (20–39), Middle Aged (40–59), Seniors (60–79), and Aged (80–99); there are no survivors beyond 99. It is convenient to refer to each age group and each corresponding time interval as a *generation*. All Children are born to the generation of Young Adult women; the fertility rate for that group is thus identical to the total fertility rate. Labour force participation is confined to the Young Adult, Middle Aged, and (in much lesser degree) Senior age groups; Children and the Aged have no participation.

Time in Alpha is measured in generations indexed by t. The population at $t = 0$ has an important characteristic, a "bulge" in the age distribution inherited from an earlier period of very high fertility — a "baby boom". The "baby boom" occurred roughly two generations earlier (at $t = -2$) and was followed by a "baby bust" — a sharp reduction in fertility and a subsequently maintained low level. The Children of the boom are in Middle Age at $t = 0$, and a generation later they will be Seniors. The population is aging.

Alpha is closed to trade but open to immigration — indeed, there is an infinite supply of potential immigrants available, and thus the possibility of using immigration as a tool to offset what is going on in the domestic population. (Note that we are talking about immigrants as permanent additions to the population, not temporary “guest workers”.) The government can set the immigration quota — the number of immigrants to be admitted in each generation — and it can set the immigrant age distribution. What follows in this paper is a model and assessment of the longer-run implications of those choices and related considerations.

2.2 The Model

The dynamics of the population and income generation are simple. Let the column vector n stand for the population by age and sex: the first five rows are female age groups (youngest to oldest), the second five are male age groups. The progress of the population from generation *t* to generation *t*+1 can be represented as

$$n_{t+1} = Qn_t + m_{t+1} \tag{1}$$

where *m* is a vector of immigrants (with age-sex elements corresponding to those of *n*, all nonnegative) and *Q* is a 10×10 Leslie matrix (Leslie, 1945, 1948); its nonzero elements are determined by age-sex-specific survival rates, the fertility rate, and the male/female birth ratio. If there were no immigration, and all rates were constant, $n_{t+1} = Qn_t$ would hold exactly for all *t*. (The matrix is defined more precisely in the Appendix.) There is no emigration, only immigration.

The vector *m* can be separated into two components, one representing the total number of immigrants, the scalar *M*, the other their proportionate age-sex distribution, the vector $\alpha \in A$, where *A* is the set of all possible age-sex distributions. We refer to *M* as the *immigration quota*. The quota is set as a proportion *q* of what the total population would be in any given generation *without* immigration. The actual total population in generation *t*+1 is $u'n_{t+1}$, where u is a column vector of ones, and the total population as it would be if there were no immigration is $u'Qn_t$. The immigration quota is then $M_{t+1} = q\left(u'Qn_t\right)$. Making the substitutions, Equation (1) can be rewritten as

$$n_{t+1} = Qn_t + M_{t+1}\alpha = Qn_t + q\left(u'Qn_t\right)\alpha \tag{2}$$

Thus *q* and α are the policy choices for the government.

The employed labour force — or simply labour force — is determined by the population vector *n* and a vector of constant participation rates *r*, shared by both immigrants and the domestic population: thus $L = r'n$.

Output *Z* (in real terms) is generated by a constant-returns-to-scale Cobb-Douglas production function, with inputs *L* for labour and *K* for capital: in log form,

$$lnZ_t = \mu + \theta t + \beta lnK_t + \left(1-\beta\right)lnL_t \tag{3}$$

where θ is the intergenerational rate of neutral technical progress, or equivalently, total factor productivity. Investment *I* is supported by a constant saving rate γ: thus $I = S = \gamma Z$. The stock of capital is subject to a rectangular or “one horse shay” depreciation function (Hulten and Wykoff, 1981). A unit of stock is undepreciated for one generation, and is then terminated; hence $K = I = \gamma Z$, a convenient simplification for our purposes. Note that since a generation is 20 years, the rectangular depreciation function provides the same number of capital service years, namely 20, as a geometric function with an annual depreciation rate of 5 percent would provide over its infinite lifetime (1/0.05 = 20). Substituting γZ for K in equation (3) and rearranging terms allows us to rewrite the production function in the simpler form

$$lnZ_t = \varphi + gt + lnL_t \tag{4}$$

where $\varphi = (\mu + \beta ln\gamma)/(1-\beta)$ and $g = \theta/(1-\beta)$. Output Z is now seen to be proportional to labour input, and hence directly responsive to changes in the population that determine the size of the labour force. The productivity growth rate g is interpreted as a labour productivity growth rate that captures the overall effect of changes in total factor productivity.

In national accounting parlance, Z can be regarded as gross domestic product, or equivalently as gross national product, since the economy is closed in all respects except immigration. We can define $Y = (1-\gamma)Z$ as net national income (note that capital depreciation over one generation is γZ) or as consumption. But again the choice of a definition does not matter for purposes of presentation and analysis: the relevant simulation results are shown in index form, and the indexes are identical, whichever definition one chooses. We shall refer to the indexes presented in the tables below as *national income indexes.*

The simplest practical measure of economic well-being for our purposes is national income per capita, Z/N. Age distribution is ignored in this measure – the denominator is an unweighted sum over all age groups. As an experimental alternative we offer also a weighted measure in the tables, Z/N_w; children are given half-weight in the calculation of N_w in this measure to capture the idea that they consume a smaller share of income than adults. Various other measures can be constructed (we have examined several) but the overall interpretation of results would be little affected.

2.3 Some General Considerations

We calibrate the model in the next section and run a series of simulations in the ones following, resulting in a set of tables that explore the effects of immigration and related issues. First though, there are some general considerations that may be helpful in thinking about the interpretation of the model and the simulation results.

The age distribution of the population is of first-order importance. The problem in prospect is the result of a distortion of the distribution brought about by the earlier boom/bust sequence of fertility rates, and the consequent imminent decline in the proportion of people of working age. The aim of immigration policy is then to shift the distribution in a different direction by increasing the proportion of working age and decreasing the proportion in the dependency age groups. Obviously that will not be accomplished if the distribution of immigrants is the same as the domestic distribution in every generation. So the focus will be on bringing in working-age adults. But there is more to the story.

There are two groups of prime working age: Young Adults and the Middle Aged. (Seniors contribute to the labour force also but in only minor degree.) Middle Aged immigrants contribute to the labour force for one generation but then move into the (mainly) dependent Seniors group in the next, and the Aged group in the one after that. Young Adults have the policy advantage of working for two generations before moving on, but they also bear children, and thus contribute to both the working population and the dependent population. In fact, children accompanying their parents may themselves represent a considerable proportion of the immigration quota. To go a step further, the children of immigrants are dependents initially but a generation later they will be in the labour force, and bearing their own children; three generations later they will be of retirement and dependency age, and so it goes.

There is also the question of how high to set the quota — how many immigrants to admit in any period. It may be theoretically possible to effect a major shift in population age distribution by setting the quota very high but practical constraints are prohibitive. There are limits to how many newcomers can be absorbed into the society without disruptive effects in any one generation. The question then is how much beneficial effect on the economy can be expected from a realistic quota, given the choice of immigration age distribution. We experiment with alternative combinations of age distribution and quota size.

2.4 Calibration and Notation

A characteristic of the Alphan population is that it is the same at generation $t = 0$ as the 2001 Canadian census population, and thus exhibits the same distorted age distribution and evidence of population aging (Statistics Canada, 2013b). Moreover: the age-sex-specific survival rates incorporated into the Q matrix are identical to Canadian rates, and can be calculated directly from the 2001 Canadian life tables; the initial (total) fertility rate of 1.6 children per woman is the Canadian rate in 2011; and the ratio of male to female births, set at 1.05, is approximately the longstanding Canadian ratio. (We emphasize that the use of Canadian demographic data for calibration is simply a matter of convenience. We take advantage of the fact that Canada provides a good example of a developed country with a "population aging problem", but we are certainly not attempting to model the Canadian economy, population dynamics, or immigration patterns and policy. See the Appendix for details and references.)

The age-sex labour force participation rates — the proportions of (employed) labour force in the population, the elements of the vector r — are roughly consistent (in broad pattern) with Canadian rates in the decade centered on 2001, with the qualifications that the rates for Children are zero and the rates for Young Adults and Middle Aged are equal. The rates for females, the top half of r, are (0, 0.75, 0.75, 0.10, 0); the rates for males, the bottom half of r, are (0, 0.85, 0.85, 0.20, 0).

Since output Z is proportional to labour input, and results are shown only as indexes, there is no need to set values for φ or the underlying β, γ, μ and θ parameters (Equation (4)). The rate of growth of productivity, g is set to zero in the initial simulations, but allowed to vary in some later ones.

The simulations involve runs with different immigrant age distributions and some simple notation is helpful in presenting results. First, note that all simulations assume that immigrants in each age group are equally divided between males and females; we do not experiment with differences in sex composition. This cuts to five the number of values that would have to be reported in defining a distribution. Moreover, we assume in most cases (Table 1 is an exception) that immigration policy choices are restricted to Children, Young Adults, and the Middle Aged; no Seniors or Aged immigrants are permitted since immigrants in those age groups would simply add to the numbers of dependents (aside from a small proportion of Seniors who enter the labour force). Our focus is on immigration as a policy device for influencing the economy, and offsetting the effects of domestic population aging. Permitting older immigrants to enter might be considered desirable for other reasons but its effect on immigration as an economic policy tool would be to weaken it. A practical result of this exclusion for presentation purposes is that the number needed to be reported in defining an immigration age distribution is now reduced to three. We choose the symbol AGEIM to stand for "age distribution of immigrants" and report the proportions in percentage form. AGEIM (25, 50, 25), for example, means that immigrants are distributed as 25 percent Children, 50 percent Young Adults, and 25 percent Middle Aged.

3. Results

3.1 Initial Simulations

We begin, in Table 1, with some simulations that exclude or include immigration. The starting population (t = 0) is shown in the first column of figures. The next three show the evolution of the population over three generations, assuming no immigration. The final three introduce immigration and trace the evolution again, assuming three alternative immigration quotas, each coupled with an age distribution identical to that of the initial (t = 0) population.

When there is no immigration the population increases by 4.5 percent in the first generation and decreases thereafter; in fact, with the fertility rate constant at 1.6 children per woman the population would decline from generation to generation indefinitely. (The

Table 1. Simulations with and without immigration; immigrants distributed by age as in the initial population

		No immigration			AGEIM like initial population		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
					---- q = 10% ----		
Population	100.0	104.5	98.0	87.1	114.9	119.8	119.7
- growth rate	—	4.5	–6.2	–11.1	14.9	4.3	–0.1
- proportion old	16.8	26.6	31.5	31.9	25.7	29.7	30.0
- proportion child	25.7	21.9	20.2	20.3	22.3	20.9	21.0
LF/Pop. ratio	48.1	44.5	42.2	41.7	44.8	43.0	42.6
National income	100.0	96.8	86.1	75.6	107.2	107.1	106.0
- per capita	100.0	92.6	87.9	86.8	93.3	89.4	88.6
- wtd. per capita	100.0	90.7	85.2	84.2	91.5	87.0	86.2
					---- q = 20% ----		
Population	100.0	104.5	98.0	87.1	125.4	143.8	159.3
- growth rate	—	4.5	–6.2	–11.1	25.4	14.7	10.8
- proportion old	16.8	26.6	31.5	31.9	25.0	28.3	28.5
- proportion child	25.7	21.9	20.2	20.3	22.5	21.4	21.5
LF/Pop. ratio	48.1	44.5	42.2	41.7	45.1	43.5	43.2
National income	100.0	96.8	86.1	75.6	117.7	130.3	143.4
- per capita	100.0	92.6	87.9	86.8	93.9	90.6	90.0
- wtd. per capita	100.0	90.7	85.2	84.2	92.2	88.4	87.9
					---- q = 30% ----		
Population	100.0	104.5	98.0	87.1	135.8	170.0	206.8
- growth rate	—	4.5	–6.2	–11.1	35.8	25.2	21.6
- proportion old	16.8	26.6	31.5	31.9	24.3	27.1	27.3
- proportion child	25.7	21.9	20.2	20.3	22.8	21.8	21.9
LF/Pop. ratio	48.1	44.5	42.2	41.7	45.4	44.0	43.8
National income	100.0	96.8	86.1	75.6	128.1	155.7	188.3
- per capita	100.0	92.6	87.9	86.8	94.4	91.5	91.1
- wtd. per capita	100.0	90.7	85.2	84.2	92.8	89.5	89.1

Note: Population and income variables are indexes; all other variables are percentages. Proportion old is percentage of Seniors and Aged combined; wtd. per capita income assigns half weights to children. The initial population age distribution is (25.7, 29.2, 28.3, 13.8, 3.0).

natural replacement rate is approximately 2.07; we experiment with different rates in later simulations.) The proportion of old people (Seniors plus Aged) increases from 16.8 percent at t = 0 to 26.6 percent at t = 1, and then almost doubles the initial level, rising to 31.5 and 31.9 percent. Concomitantly, the proportion of Children decreases. The ratio of labour force to population falls from 48.1 percent at t = 0 to 44.5 percent at t = 1, and then to 42.2 and 41.7 percent, producing sharp declines in the national income index: from a base of 100.0 at t = 0, income falls to 96.8 at t = 1, 86.1 at t = 2, and 75.6 at t = 3. Income per capita falls accordingly, but less precipitously after one generation, since the population is also declining: the unweighted measure falls to 92.6, 87.9, and 86.8; the weighted measure falls even more — to 90.7, 85.2, and 84.2. Such is the population/economy trajectory in the absence of immigration. We have run the simulations out for several more generations beyond the three for which results are shown in the table but the longer-run pattern is clear after three: a continuing high proportion of old people relative to the base generation, a continuing lower proportion of children, a much reduced labour force-to-population ratio, a declining national income, and a much lower level of income per capita, weighted or unweighted.

Immigration is introduced in Table 1 (and in subsequent tables) at three quota levels: 10, 20, and 30 percent per generation. (The corresponding annual rates are approximately 0.48, 0.92, and 1.32 percent; a sustained level of .48 would be considered rather high by modern international standards for an industrialized country, and 1.32 as very high.) As noted above, the age distribution chosen for this first set of simulations with immigration is the distribution of the population as it was at t = 0; it is chosen simply as a reference case. One effect is to stop the decline of the population (with the exception of a very slight dip when q = 10 percent, at t = 3). The proportion of old people is a little lower than in the no-immigration case and the labour force/population ratio a little higher, although it takes a very high quota rate to have much effect in that regard. The immediate decline of national income is arrested: with q = 10 percent income roughly levels off; it increases significantly with q = 20 percent and rapidly with q = 30 percent. But income per capita (either measure) never recovers; it is higher than the corresponding no-immigration level in all cases but still well below what is was at t = 0. In short, bringing in immigrants with the base level age distribution can moderate the income decline induced by population aging, but only in limited degree if one takes account of the effect of immigration on the size of the population as well as the level of economic activity, and then only with a high quota level.

3.2 Immigration with Working-age Concentration

Choosing an age distribution with a high concentration of immigrants in the working ages — Young Adults and Middle Aged — makes a big difference. Table 2 assumes two such distributions: (a) 50 percent Young Adults, 25 percent Middle Aged, plus 25 percent Children; (b) 75 percent Young Adults, no Middle Aged, plus 25 percent Children. Both distributions raise the labour force/population ratio and increase the level of national income per capita (either measure) above what it would have been had there been no immigration, and also above the level resulting from the immigrant age distribution assumed in Table 1. The effects are greater, the higher the quota. The immediate effect (t = 1) is the same for both distributions but by the second generation (t = 2) the Middle Aged immigrants admitted previously under distribution (a) have become Seniors, and thus started to add to the dependent population. Under distribution (b) this effect is delayed until the third generation (t = 3).

A fraction of the decline in income per capita from the base period is offset under either distribution. The quota matters greatly in this regard but whatever the quota, the distribution

Table 2. Simulations with alternative immigrant age distributions when there are child immigrants

		AGEIM (25, 50, 25)			AGEIM (25, 75, 0)		
	t = 0	*t* = 1	*t* = 2	*t* = 3	*t* = 1	*t* = 2	*t* = 3
				q = 10%			
Population	100.0	114.9	123.2	127.1	114.9	125.9	134.3
- growth rate	—	14.9	7.2	3.2	14.9	9.5	6.7
- proportion old	16.8	24.2	26.8	27.5	24.2	24.5	25.4
- proportion child	25.7	22.2	21.7	21.3	22.2	22.9	22.0
LF/Pop. ratio	48.1	45.9	44.3	44.1	45.9	44.8	45.0
National income	100.0	109.8	113.5	116.7	109.8	117.3	125.7
- per capita	100.0	95.5	92.1	91.8	95.5	93.2	93.7
- wtd. per capita	100.0	93.7	90.1	89.5	93.7	91.8	91.7
				q = 20%			
Population	100.0	125.4	151.1	177.4	125.4	157.0	194.6
- growth rate	—	25.4	20.5	17.4	25.4	25.2	24.0
- proportion old	16.8	22.2	23.3	24.1	22.2	19.6	20.7
- proportion child	25.7	22.4	22.8	22.1	22.4	24.7	23.4
LF/Pop. ratio	48.1	47.1	45.9	45.9	47.1	46.7	47.2
National income	100.0	122.8	144.3	169.4	122.8	152.6	191.2
- per capita	100.0	98.0	95.5	95.5	98.0	97.2	98.3
- wtd. per capita	100.0	96.2	93.9	93.6	96.2	96.7	97.0
				q = 30%			
Population	100.0	135.8	181.8	238.9	135.8	191.3	269.7
- growth rate	—	35.8	33.9	31.4	35.8	40.9	40.9
- proportion old	16.8	20.5	20.5	21.4	20.5	16.1	17.3
- proportion child	25.7	22.6	23.5	22.7	22.6	25.9	24.4
LF/Pop. ratio	48.1	48.1	47.2	47.3	48.1	48.2	48.8
National income	100.0	135.9	178.7	235.1	135.9	192.1	274.0
- per capita	100.0	100.1	98.3	98.4	100.1	100.4	101.6
- wtd. per capita	100.0	98.3	97.0	96.8	98.3	100.5	100.8

Note: See relevant parts of note to Table 1.

with the higher proportion of Young Adults dominates. However, even that one requires a high quota to eliminate the decline from the initial level; to come close requires a quota of 20 percent, to eliminate the decline entirely requires a quota of 30 percent, and even then the result is not achieved until the second generation.

3.3 The Effect of Eliminating Child Immigrants

Child immigrants augment immediately the dependent component of the population and it is of interest therefore to explore the consequences of restricting admission to adults. The two immigration choices in Table 3 repeat the distributions of adult immigrants in Table 2 but now stipulate no Child immigrants; the quotas remain the same but the immigration totals consist entirely of adults. The effects are immediate and significant. The income per capita indexes are higher than they were with Children included, in all cases, and the decline from base level is eliminated, all but eliminated, or even converted to an increase with quotas of 20 and 30 percent coupled with the most highly concentrated of the two adult age distributions. Exact results depend on whether one uses the weighted or

Table 3. Simulations with alternative immigrant age distributions when there are no child immigrants

		AGEIM (0, 67, 33)			AGEIM (0, 100, 0)		
	t = 0	*t* = 1	*t* = 2	*t* = 3	*t* = 1	*t* = 2	*t* = 3
	---------- *q* = 10% ----------						
Population	100.0	114.9	124.5	127.1	114.9	128.1	136.6
- growth rate	—	14.9	8.4	2.1	14.9	11.5	6.7
- proportion old	16.8	24.2	27.1	29.4	24.2	24.1	26.5
- proportion child	25.7	19.9	20.4	18.6	19.9	21.9	19.7
LF/Pop. ratio	48.1	47.7	45.2	45.0	47.7	45.9	46.2
National income	100.0	114.1	117.1	118.9	114.1	122.3	131.3
- per capita	100.0	99.3	94.1	93.5	99.3	95.5	96.1
- wtd. per capita	100.0	96.1	91.3	89.9	96.1	93.5	92.9
	---------- *q* = 20% ----------						
Population	100.0	125.4	154.1	178.1	125.4	161.8	201.4
- growth rate	—	25.4	22.9	15.6	25.4	29.1	24.4
- proportion old	16.8	22.2	23.7	26.8	22.2	19.1	22.1
- proportion child	25.7	18.3	20.1	17.6	18.3	22.5	19.4
LF/Pop. ratio	48.1	50.4	47.8	47.7	50.4	48.9	49.5
National income	100.0	131.5	153.2	176.7	131.5	164.7	207.5
- per capita	100.0	104.9	99.4	99.2	104.9	101.8	103.0
- wtd. per capita	100.0	100.6	96.3	94.8	100.6	99.9	99.4
	---------- *q* = 30% ----------						
Population	100.0	135.8	186.6	241.5	135.8	199.3	283.3
- growth rate	—	35.8	37.4	29.4	35.8	46.7	42.2
-proportion old	16.8	20.5	21.1	24.5	20.5	15.5	18.7
- proportion child	25.7	16.9	19.5	16.8	16.9	22.4	19.1
LF/Pop. ratio	48.1	52.7	50.0	50.0	52.7	51.4	52.1
National income	100.0	148.9	194.3	251.2	148.9	213.2	307.3
- per capita	100.0	1O9.7	104.1	104.0	109.7	107.0	108.5
- wtd. per capita	100.0	104.4	100.5	99.0	104.4	105.0	104.6

Note: See relevant parts of note to Table 1.

unweighted per capita measure for comparison but the general nature of the effects is clear: excluding Child immigrants raises per capita national income above what it would otherwise have been, both immediately and in subsequent generations.

3.4 The Implications of Quota/age Distribution Choices: A Closer Look

The choice of a quota establishes the total number of immigrants in any generation as a proportion of the population, calculated as it would be if there were no immigrants. We experiment with three quotas, 10, 20, and 30 percent. Policy makers would have to judge whether these quotas were acceptable in relation to the overall size of the population or whether they would pose difficulties in absorbing the resulting numbers of new immigrants into the society. But the choice of an age distribution takes the absorption issue further; it invites the question of whether the implied number of immigrants *in each age group* is acceptable. We consider now, from that point of view, the number of immigrants as a proportion of the population in each group. We do this for generation 1 and show the results in Table 4.

Referring back to Section 2.2, the total population in generation 1 can be obtained from Equation (2) as

$$N_1 = u'n_1 = u'n_0 + q\left(u'Qn_0\right) \quad (5)$$

where u is again a vector of ones. Let b_1 be the vector of age-sex proportions of the overall population in generation 1 (corresponding to α, the age-sex proportions vector for immigrants). We may then write

$$n_1 = N_1 b_1 = \left(u'Qn_0 + q\left(u'Qn_0\right)\right)b_1 \quad (6)$$

The immigration total is $M_1 = q\left(u'Qn_0\right)\alpha$. Letting $diag\left(\alpha\right)$ and $diag(b_1)$ be diagonal matrices in which α and b_1 are the diagonals, we write

$$\begin{aligned} H &= \left(q\left(u'Qn_0\right)diag\left(\alpha\right)\right)(u'Qn_0 + q\left(u'Qn_0\right)diag(b_1))^{-1} \\ &= (q/\left(1+q\right))diag\left(\alpha\right)diag(b_1)^{-1} \end{aligned} \quad (7)$$

The age-sex-specific immigrant proportions are the diagonal elements of H and the overall share proportion is $M_1 / N_1 = q/\left(1+q\right)$. Age-specific (male plus female) share proportions based on Equation (7) are shown in Table 4 for the three immigration quotas and the alternative age distributions used in the earlier tables.

Age distributions with concentrations in the working age groups can increase markedly the level of national income per capita, as shown in Tables 2 and 3. But a concomitant of that is a high proportion of immigrants in those particular groups and possible difficulties

Table 4. Immigrants in generation 1 as percentage of population, by age group, based on alternative choices of immigration quota and age distribution

	Immigration age distribution (AGEIM)				
	Like initial pop.	(25, 50, 25)	(25, 75, 0)	(0, 67, 33)	(0, 100, 0)
	---------- $q = 10\%$ ----------				
Children	10.5	10.2	10.2	0.0	0.0
Young Adults	10.7	17.0	23.5	21.6	29.1
Middle Aged	9.5	8.4	0.0	10.9	0.0
Seniors	5.8	0.0	0.0	0.0	0.0
Aged	6.7	0.0	0.0	0.0	0.0
All ages	9.1	9.1	9.1	9.1	9.1
	---------- $q = 20\%$ ----------				
Children	19.0	18.6	18.6	0.0	0.0
Young Adults	19.3	29.1	38.1	35.4	45.0
Middle Aged	17.3	15.5	0.0	19.6	0.0
Seniors	11.0	0.0	0.0	0.0	0.0
Aged	12.5	0.0	0.0	0.0	0.0
All ages	16.7	16.7	16.7	16.7	16.7
	---------- $q = 30\%$ ----------				
Children	26.0	25.5	25.5	0.0	0.0
Young Adults	26.4	38.1	47.9	45.2	55.2
Middle Aged	23.8	21.7	0.0	26.7	0.0
Seniors	15.6	0.0	0.0	0.0	0.0
Aged	17.3	0.0	0.0	0.0	0.0
All ages	23.1	23.1	23.1	23.1	23.1

of absorbing the implied large numbers of newcomers of a given age into the society. The issue of absorption lies outside our model framework but it is something that the government would have to consider. The extreme situations in both national income benefits and possible absorption difficulties occur when only Young Adult immigrants are admitted to the country — the distribution (0, 100, 0). With a quota of 10 percent, 29 percent of the population in that age group are immigrants; with a 20 percent quota, 45 percent are immigrants; and with a 30 percent quota the proportion is well over half, 55 percent. Even with the somewhat less concentrated (0, 67, 33) distribution the proportion in the Young Adult age group reaches 35 percent with a 20 percent quota and 45 percent with a quota of 30 percent. The policy choice that the government must make poses a tradeoff — accepting a lower level of income per capita than what might be attainable through immigration vs. possible societal absorption difficulties with a higher immigration quota.

3.5 Productivity Growth as an Offset to Population Aging

The rate of growth of productivity is denoted by g in Equation (4), Section 2.2. We have set g to zero in all of the simulations thus far. Now we experiment with positive values. The immigration quota and age distribution are under government control; the rate of productivity growth is not. The government may be able to nudge the rate a little by this or that policy but the extent of its influence is no doubt limited. Nevertheless, it is of interest to see how productivity growth might act as an offset to the negative effect of population aging on the economy.

Table 5 shows what would happen to national income per capita (unweighted) if a productivity growth rate of 5 or 10 percent were coupled with an immigration quota of 0, 10, 20, or 30 percent, using the (25, 50, 25) age distribution for the calculations in these experiments. (A productivity growth rate of 5 percent per generation is equivalent to an annual rate of only 0.24 percent; a growth rate of 10 percent per generation is equivalent to an annual rate of 0.48 percent.)

The results in Table 5 appear striking: productivity growth of 10 percent per generation

Table 5. Simulations of national income per capita assuming alternative rates of productivity growth (g), with and without immigration (q)

	$t = 0$	$t = 1$	$t = 2$	$t = 3$
$q = 0$ (no immigration)				
$g = 0$	100.0	92.6	87.9	86.8
$g = 5\%$	100.0	97.2	96.9	100.5
$g = 10\%$	100.0	101.9	106.3	115.5
$q = 10\%$				
$g = 0$	100.0	95.5	92.1	91.8
$g = 5\%$	100.0	100.3	101.6	106.2
$g = 10\%$	100.0	105.1	111.5	122.1
$q = 20\%$				
$g = 0$	100.0	98.0	95.5	95.5
$g = 5\%$	100.0	102.9	105.3	110.5
$g = 10\%$	100.0	107.8	115.6	127.1
$q = 30\%$				
$g = 0$	100.0	100.1	98.3	98.4
$g = 5\%$	100.0	105.1	108.3	113.9
$g = 10\%$	100.0	110.1	118.9	131.0

Note: AGEIM is (25, 50, 25) in all cases where there is immigration.

by itself, with no immigration, would wipe out immediately (t = 1) the decline of national income per capita brought about by population aging, and raise the income per capita level further in subsequent generations. Coupling even a 5 percent growth rate with positive immigration quotas would set an upward trajectory for income per capita. It would seem then that even a modest rate of productivity growth would eliminate all concerns about the economic effects of population aging. However, that interpretation is superficial.

Suppose, to make a point, that while the productivity growth rate in Alpha is 5 percent, the growth rate in the rest of the world is 10 percent. *Relative to other countries* Alpha's national income per capita would then fall by about 4.5 percent in the first generation (on top of whatever was the decline resulting from population aging), by 8.9 percent in the second, and so on. The point is that to be interpreted realistically, the productivity growth rate should be defined as the difference from the growth rate in the rest of the world. Moreover, if the economy were open rather than closed it would find its terms of trade deteriorating and its relative standard of living falling as a result of its slower productivity growth. If g is defined as a *differential* rate of productivity growth, a positive rate would indeed offset some or all of the effects of population aging on the economy. Zero productivity growth, as we have assumed in the earlier simulations, would then imply that productivity was growing in Alpha at the same rate as elsewhere and that income was measured correspondingly, in relative terms.

3.6 What if the Fertility Rate were to Increase?

The "natural replacement" fertility rate is a little under 2.1 children per woman. That is the rate required for the population to achieve a stationary state in the long run — constant population size and an unchanging age distribution. A higher rate would result in continuous population increase, a lower rate in continuous population decline. What if the rate were to increase from the 1.6 level assumed up to now?

Letting F stand for fertility rate, we experiment with two higher levels, starting at t = 1: the levels are F = 2.0745 (the natural replacement rate to four decimal places) and F = 2.5, a value well above replacement. Would such higher rates add to or diminish the effects of population aging on the economy?

The results of the experiments are presented in Table 6. To isolate fertility effects we assume no immigration. The top panel of the table repeats the no-immigration results from Table 1, with the fertility rate held at 1.6. The middle and bottom panels show results for the two higher rates.

With F equal to the replacement rate, the population increases more rapidly at t = 1, and remains at the higher level thereafter, thus arresting the long-run population decline observed previously. But a higher value of F means more children in the first generation, more dependents in the population, a lower labour force/population ratio, and a lower level of national income per capita. The unweighted per capita income index has dropped significantly, from 92.6 (when the fertility rate was 1.6) to 87.0 with the new higher rate; the weighted index has dropped somewhat less, from 90.7 to 87.5. In the second generation (t = 2) the children of the first have come of working age but a new cohort of child dependents has taken their place, and there are only small changes in the labour force/population ratio and per capita income indexes. There are some further differences in generation 3 but overall the picture is generally similar to that of generation 2.

Much the same can be said, qualitatively, for the results of the further increase in fertility rate to 2.5. What were smaller effects with replacement fertility though have now become bigger ones. Most notably, the reduction of per capita income (weighted or unweighted) in the first generation is much greater.

In sum, the effect on the economy of an increase in the fertility rate in the first generation

Table 6. Simulations assuming alternative fertility rates (*F*) with no immigration

	t = 0	*t* = 1	*t* = 2	*t* = 3
		F = 1.6		
Population	100.0	104.5	98.0	87.1
- growth rate	—	4.5	-6.2	-11.1
- proportion old	16.8	26.6	31.5	31.9
- proportion child	25.7	21.9	20.2	20.3
LF/Pop. Ratio	48.1	44.5	42.2	41.7
National income	100.0	96.8	86.1	75.6
- per capita	100.0	92.6	87.9	86.8
- wtd. per capita	100.0	90.7	85.2	84.2
		F = 2.0745		
Population	100.0	111.3	110.6	111.4
- growth rate	—	11.3	-0.6	0.8
- proportion old	16.8	25.0	27.9	25.0
- proportion child	25.7	26.7	23.2	26.6
LF/Pop. ratio	48.1	41.8	42.3	41.5
National income	100.0	96.8	97.3	96.1
- per capita	100.0	87.0	88.0	86.3
- wtd. per capita	100.0	87.5	86.7	86.7
		F = 2.5		
Population	100.0	117.3	121.9	135.9
- growth rate	—	17.3	3.9	11.5
- proportion old	16.8	23.7	25.3	20.5
- proportion child	25.7	30.5	25.4	31.7
LF/Pop. ratio	48.1	39.6	42.3	40.5
National income	100.0	96.8	107.4	114.6
- per capita	100.0	82.4	88.1	84.3
- wtd. per capita	100.0	84.8	87.9	87.3

Note: See relevant parts of note to Table 1. *F* = 2.0745 is the natural replacement rate.

can be large and unfavourable, from the point of view of income per capita, owing to the addition of more child dependents. The effects in the subsequent generations, when the earlier-generation children have entered the labour force, are smaller, and somewhat mixed.

3.7 Effects of Reduced Mortality and Increased Participation of Seniors

The simulations to this point have assumed constant mortality and labour force participation rates. We experiment now with declining mortality rates, considered alone and in combination with increased participation of seniors, both with and without concurrent immigration. The mortality assumption is that age-sex death rates would decline over the next three generations at the same average proportionate rates of change as in the last three. (These rates of change are the Canadian life table rates calculated over the 60-year period 1941–2001; see Dominion Bureau of Statistics, 1947, and Statistics Canada, 2006, for the life tables used in the calculations.) The participation assumption is that participation rates of Seniors would increase by half in the first generation, and stay at the new levels in the subsequent two; that means that the participation rate for males would increase from 20 percent to 30 percent, the rate for females from 10 percent to 15 percent. The assumptions about accompanying immigration are a 20 percent quota and a (25, 50, 25) age distribution. The results of the experiments are presented in Table 7. The top panel in the table repeats results from Tables 1 and 2: constant mortality rates are assumed, with and without immigration. The middle panel assumes declining mortality and the bottom one declining mortality plus increased participation rates, with and without immigration in both cases.

The most prominent effect of declining mortality, taken alone, is to increase the proportion of older dependents in the population, decrease the labour force/population ratio, and lower both measures of income per capita. Immigration operates in the opposite direction, and much more strongly. Introducing increased participation of seniors in the bottom panel of the table offsets the increased dependency effect of lower death rates and has a net positive effect on income per capita, but immigration is again the dominant contributor. In short, declining mortality lowers per capita income, declining mortality plus increasing Seniors' participation rates by half raises it, but while the net effect is significant it takes second place to the effect of immigration.

Table 7. Simulations with and without immigration, allowing for declining mortality rates and increased labour force participation of Seniors

		No immigration			Immigration, q = 20%		
	t = 0	t = 1	t = 2	t = 3	t = 1	t = 2	t = 3
	constant mortality						
Population	100.0	104.5	98.0	87.1	125.4	151.1	177.4
- growth rate	—	4.5	–6.2	–11.1	25.4	20.5	17.4
- proportion old	16.8	26.6	31.5	31.9	22.2	23.3	24.1
- proportion child	25.7	21.9	20.2	20.3	22.4	22.8	22.1
LF/Pop. ratio	48.1	44.5	42.2	41.7	47.1	45.9	45.9
National income	100.0	96.8	86.1	75.6	122.8	144.3	169.4
- per capita	100.0	92.6	87.9	86.8	98.0	95.5	95.5
- wtd. per capita	100.0	90.7	85.2	84.2	96.2	93.9	93.6
	declining mortality						
Population	100.0	106.5	103.3	94.7	127.8	158.8	192.6
- growth rate	—	6.5	–3.0	–8.4	27.8	24.2	21.3
- proportion old	16.8	27.6	34.3	36.3	23.0	25.3	27.0
- proportion child	25.7	21.5	19.3	18.8	22.1	22.0	21.1
LF/Pop. ratio	48.1	44.1	40.9	39.5	46.7	45.0	44.5
National income	100.0	97.7	87.8	77.8	124.3	148.6	178.3
- per capita	100.0	91.7	85.0	82.2	97.2	93.6	92.6
- wtd. per capita	100.0	89.6	82.0	79.1	95.3	91.7	90.2
	declining mortality, increased LFP						
Population	100.0	106.5	103.3	94.7	127.8	158.8	192.6
- growth rate	—	6.5	–3.0	–8.4	27.8	24.2	21.3
- proportion old	16.8	27.6	34.3	36.3	23.0	25.3	27.0
- proportion child	25.7	21.5	19.3	18.8	22.1	22.0	21.1
LF/Pop. ratio	48.1	45.8	42.7	41.3	48.1	46.4	46.0
National income	100.0	101.4	91.7	81.4	128.0	153.2	184.2
- per capita	100.0	95.2	88.8	86.0	100.1	96.5	95.6
- wtd. per capita	100.0	93.0	85.6	82.7	98.1	94.5	93.2

Note: See relevant parts of note to Table 1. AGEIM is (25, 50, 25) when there is immigration. Declines in mortality are at the average group-specific percentage rates of decrease per generation over the previous three-generation time span. Increased LFP means Seniors' labour force participation rates are increased by half (from 20% to 30% for men, 10% to 15% for women).

3.8 Effects of Differential Education Assumptions

This last set of simulations, reported in Table 8, explores the effects of assuming different education levels for immigrants in conjunction with alternative age distributions, and the consequent effects on productivity and income. There are three assumptions as follows:

(i) Immigrants have the same education and hence the same productivity level as the domestic labour force (the assumption in earlier simulations); an additional immigrant member of the labour force thus increases output in the same proportion as an additional domestic member.

(ii) Immigrants have a higher level of education with the result that their productivity is 20% greater than domestic productivity.

(iii) Immigrants have a lower level of education with the result that their productivity is 20% lower than domestic productivity. The quota q is set at 20% in all cases and the five immigrant age distributions defined previously are coupled separately with each of these three assumptions. The alternative productivity levels are applied in each period to the new immigrants of that period and the surviving immigrants of previous periods.

The impact on national income of a higher level of education-related productivity is seen immediately, in period 1, and again in the subsequent two periods. The impact translates also into higher per capita income levels. (The increase in immigrant productivity is equivalent to an increase in the size of the immigrant labour force with no corresponding increase in the consuming population.) The magnitude of the effects differs with the assumption about the immigrant age distribution — greater for distributions with higher concentrations in the working ages, lower for others.

Table 8. Simulations when education-related immigrant productivity can be the same, higher, or lower than domestic productivity, with alternative immigration age distributions and q = 20%

		Same productivity			Productivity higher by 20%			Productivity lower by 20%		
	$t = 0$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
		AGEIM like initial population								
National income	100.0	117.7	130.3	143.4	121.9	139.1	155.3	113.5	121.4	131.4
- per capita	100.0	93.9	90.6	90.0	97.2	96.7	97.5	90.5	84.4	82.5
- wtd per capita	100.0	92.2	88.4	87.9	95.5	94.4	95.2	88.9	82.4	80.5
		AGEIM (25,50,25)								
National income	100.0	122.8	144.3	169.4	128.1	156.0	185.4	117.6	132.7	153.4
- per capita	100.0	98.0	95.5	95.5	102.1	103.2	104.5	93.8	87.8	86.5
- wtd per capita	100.0	96.2	93.9	93.6	100.3	101.5	102.4	92.1	86.4	84.7
		AGEIM (25,75,0)								
National income	100.0	122.8	152.6	191.2	128.1	166.0	210.3	117.6	139.3	172.2
- per capita	100.0	98.0	97.2	98.3	102.1	105.7	108.0	93.8	88.8	88.5
- wtd per capita	100.0	96.2	96.7	97.0	100.3	105.1	106.6	92.1	88.3	87.3
		AGEIM (0,67,33)								
National income	100.0	131.5	153.2	176.7	138.5	166.6	193.2	124.6	139.7	160.1
- per capita	100.0	104.9	99.4	99.2	110.5	108.1	108.5	99.4	90.7	89.9
- wtd per capita	100.0	100.6	96.3	94.8	106.0	104.7	103.6	95.3	87.9	85.9
		AGEIM (0,100,0)								
National income	100.0	131.5	164.7	207.5	138.5	180.4	228.4	124.6	149.0	186.6
- per capita	100.0	104.9	101.8	103.0	110.5	111.5	113.4	99.4	92.1	92.7
- wtd per capita	100.0	100.6	99.9	99.4	106.0	109.5	109.5	95.3	90.4	89.4

Note: See relevant parts of note to Table 1.

Going the other way — reducing immigrant productivity by 20% — produces similar results, but in the opposite direction, again depending on the assumed age distribution. We interpret these results as implying a lower education level among immigrants. An alternative interpretation though would be that the education level is the same but the lower productivity reflects a higher rate of immigrant unemployment, or that there is underemployment — employment of immigrants in jobs beneath their education level. Any of these interpretations, or any combination, would have the same effect — a reduction of the productivity of the immigrant labour force.

We consider only overall income per capita in relation to education and productivity, not the actual distribution of income between immigrants and the domestic population. However, if immigrants were to have a higher marginal product than domestic workers but receive the same per capita income there would be implicit discrimination — in effect a transfer of wealth created from the immigrant to the non-immigrant population. If immigrants were to have a lower marginal product there would be a transfer in the opposite direction. (More generally, the education characteristics of immigrants and their implications for the economy, and for the immigrants themselves, is a topic on its own that deserves greater attention than we are able to give it here.)

4. Conclusions

Our mythical country of Alpha faces a problem common to many industrialized countries: a shift in the age distribution of the population towards a lower proportion in the labour force and consequent downward pressure on national income per capita. Immigration can be used to moderate the shift but to be effective the quota level may have to be high, the distribution of adult immigrants highly concentrated in the working ages, and the proportion of child immigrants low. While immigration will bring about an increase in aggregate national income it will also add to the number of consumers sharing in the increase. The worker/dependent ratio among immigrants is therefore a fundamental consideration in policy design. A larger quota will of course produce a larger effect but how large a quota is acceptable from a social point of view is another fundamental consideration. A higher overall level of productivity could offset the aging-induced decline in per capita income but to be realistically interpreted, productivity would have to be defined in relative terms — relative to the level in the rest of the world, that is. A higher level of education and hence productivity for immigrants alone would increase overall per capita income, a lower level would decrease it, and in either case there is the issue of how the difference would be shared between immigrants and non-immigrants. An increase in fertility would raise the proportion of dependents in the population and lower per capita income, both immediately and subsequently. Falling death rates and rising life expectancy would increase the proportion of older dependents; that could be offset by higher labour force participation rates of older people but the increases would have to be proportionately large, and even then might provide only a modest contribution.

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Ethics Statement

The analyses described in this paper were performed using secondary data obtained from publicly available sources as outlined in the Data and Methods section.

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Appendix: The Leslie Matrix

The Leslie matrix Q used in Equation (1) and subsequent equations is the 10×10 matrix shown in Table A1. The first five rows are for female age groups, youngest to oldest; the next five rows are for males. The entry in the Q(1,2) cell represents the calculation of female children, incorporating an adjustment for newborn mortality: F is the fertility rate (applied to Young Adult females), r_f is the proportion of females at birth, and s_{f0} is the survival rate for female births; the entry in the Q(6,2) cell, $s_{m0}r_mF$, represents the corresponding calculation for male children. The group-to-group survival rates for females are provided in cells Q(2,1), Q(3,2), Q(4,3), Q(5,4); the corresponding rates for males are provided in cells Q(7,6), Q(8,7), Q(9,8), Q(10,9).

The Q matrix can be applied sequentially to project an initial population vector n_0 k generations ahead, ignoring immigration and assuming all rates constant: $n_1 = Qn_0, n_2 = Qn_1, \cdots, n_k = Qn_{k-1}$ or, more compactly, $n_k = Q^k n_0$. (For discussion of Leslie matrices, their characteristics and application, see Keyfitz and Caswell, 2005, Chapter 7.)

The survival rates in Q are calibrated using 2001 Canadian life tables. (The tables are based on deaths in the years 2000, 2001, 2002 but are commonly referred to as 2001 tables (Statistics Canada, 2006). F is initially set at 1.6 children per woman, the total fertility rate in Canada in 2011 (Statistics Canada, 2013a). The ratio of males to females at birth is set at 1.05, yielding 0.488 and 0.512 as the female and male proportions, approximately the longstanding proportions in Canada. (The ratio 1.05 is within a normal range: "In the absence of manipulation, the sex ratio at birth is remarkably consistent across human populations, with 105 – 107 male births for every 100 female births," Hesketh and Xing, 2006, p 13271.)

Table A1. The Q matrix for a stable Alpha population with calibrated survival rates

	Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10
Row 1	0	$s_{f0}r_fF$	0	0	0	0	0	0	0	0
Row 2	0.9942	0	0	0	0	0	0	0	0	0
Row 3	0	0.9769	0	0	0	0	0	0	0	0
Row 4	0	0	0.8635	0	0	0	0	0	0	0
Row 5	0	0	0	0.3798	0	0	0	0	0	0
Row 6	0	$s_{m0}r_mF$	0	0	0	0	0	0	0	0
Row 7	0	0	0	0	0	0.9875	0	0	0	0
Row 8	0	0	0	0	0	0	0.9617	0	0	0
Row 9	0	0	0	0	0	0	0	0.785	0	0
Row10	0	0	0	0	0	0	0	0	0.2575	0

Note: $s_{f0} = 0.9940$, $s_{m0} = 0.9924$.